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TRANSITIONAL NON-NEWTONIAN FLOW IN HALF ROUND CHANNELS

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Abstract: The prediction of non-Newtonian transitional flow in open channels is more complex than for pipe flow. Not much work has been done in the field. The application is in mine waste and sludge disposal where it is important to distinguish between laminar, transitional and turbulent flows. Transitional flow is often characterised by its unstable nature. Thus, it is important to predict the transition region so it can be avoided.

Three non-Newtonian fluids, namely, kaolin and bentonite suspensions and CMC solutions at different concentrations were tested in two different flumes of 10 m long by 300 mm wide and 10 m long by 150 mm wide half-round tilting flume which could be tilted up to 5 degrees from the horizontal plane. The rheology of the fluids was tested in an in-line tube viscometer with 3 tubes of diameters 13 mm, 28 mm and 80 mm. Flow depth and flow rate measurements were made in the flumes at various flow rates spanning laminar and turbulent conditions and at different slopes from 1 to 5 degrees. The relationship between Froude and Reynolds numbers were used for these predictions by Haldenwang (2003).

Haldenwang (2003) and Haldenwang et al. (2010) proposed models for predicting the onset of transition and full turbulence in rectangular channels. This method was extended to half-round channels. Fitton (2008) established a model for predicting the point of laminar-turbulent transition. After comparing the critical velocity values for both models, it was found that they were both less accurate. Fitton (2008) used the relationship between the friction factor and the Reynolds number whereas Haldenwang et al. (2010) used the relationship between the Froude number and the Reynolds number. Since the Froude number shows the onset of transition better than the friction factor, Haldenwang's (2003) approach was selected to establish new critical Reynolds numbers for the onset and end of transitional flow for half-round channels.

The transition model proposed by Fitton (2008) for half-round channels was tested and the new developed model performed better than the ones available in the literature.

KEY WORDS: Open channel flow, non-Newtonian, rheology, laminar, transition, turbulence.

NOTATIONS

Symbol	Description	S.I. Units
A	Area	m^2
B	Breadth of channel	m
D	Tube diameter	m
e	Hydraulic roughness	m
f	Fanning friction factor	-

f_D	Darcy friction factor	-
Fr	Froude number	-
g	Gravitational acceleration	m/s^2
h	Height	m
k	Fluid consistency index	$Pa.s^n$
K	Open channel shape constant	-
m	slope	-
n	Flow behaviour index	-
Re	Reynolds number	-
$Re_{c\ (turb)}$	Critical Reynolds number at onset of full turbulence	-
Re_H	Haldenwang *et al.* (2002) Reynolds number	-
R_h	Hydraulic radius	m
V	Velocity	m/s
V_c	Critical velocity	m/s

1. INTRODUCTION

Open channels find their applications in the mining, pulp and paper, and polymer processing industries (Develter and Duffy, 1998; Fuentes et al., 2002; Kozicki and Tiu, 1988 and Sanders et al., 2002). In the mining and mineral processing industries homogeneous non-Newtonian slurries have to be transported around plants and to tailings dams (Sanders et al., 2002). Slurries are mostly transported in open channels due to its economical advantages. As water becomes scarcer and more expensive due to legislative considerations, it is desirable to convey slurries of higher concentration and due to the rapid increase in their viscosity, the flow conditions tend to be laminar and/or transitional rather than the usual turbulent conditions encountered with water-like dilute slurries. Since in the transitional regime, flow tends to be inherently unstable, it is best to avoid the operation of a flume in this region. There are situations where it is not possible to avoid this flow regime. Thus, it is desirable to be able to predict the conditions for the laminar-transitional and transitional-turbulent flow regimes in a reliable manner. Naturally, this transition is expected to be influenced by the shape of the flow area in addition to the rheological properties of the slurry.

Although, the field of open channel hydraulics is complex and vast, there is one subject of great interest to engineers which is the carrying capacity of the channel (Abulnaga, 2002). The conveyance capacity of a channel would be maximum when the wetted perimeter is the least for a given area. Thus it is found based on this criterion, that the most efficient shape is the half-round, that is, the circular section running half full (Mott, 2000). The transport of slurries in channels has limitations since no pumps are used to force the flow and the gravity is the only motive force for flow to occur. There is not a single composite mathematical model in literature which can represent slurry flows in open channels (Abulnaga, 2002). The flow of homogeneous non-Newtonian fluids in half-round open channels has only received limited attention, e.g., see Fitton (2008), and Burger et al. (2010a,b).

In this paper only transition of homogeneous slurries (relatively small particles kept in suspension by water) in half-round channels will be investigated. These slurries are non-Newtonian in behaviour and can be rheologically characterised. As this work focuses on the half-round channels, the characteristics of a half-round channel are given in

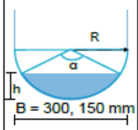
Flume shape	Cross-sectional area	Wetted perimeter	Surface width	Hydraulic radius	Froude number
Semi-circular 	$\frac{D^2}{8}(\alpha - \sin\alpha)$ where $\alpha = 2\cos^{-1}\left(1-\frac{2h}{D}\right)$	$D\left(\frac{1}{2}\alpha\right)$ where $\alpha = 2\cos^{-1}\left(1-\frac{2h}{D}\right)$	$D\left(\sin\frac{1}{2}\alpha\right)$ where $\alpha = 2\cos^{-1}\left(1-\frac{2h}{D}\right)$	$\frac{D}{4\alpha}\left[\alpha - \sin\alpha\right]$	$Fr = \frac{V}{\sqrt{g\frac{D(\alpha-\sin\alpha)}{8(\sin 0.5\alpha)}}}$

Figure 1: Half-round flume characteristics

Haldenwang et al. (2002) established a Reynolds number for non-Newtonian fluids flowing in open channels as:

$$Re_H = \frac{8\rho V^2}{\tau_y + K\left(\frac{2V}{Rh}\right)^n} \qquad (1)$$

where K, τ and n are rheological parameters of the fluid.

Solutions of CMC and suspensions of bentonite and kaolin clay which are time-independent were characterised by their rheological behaviour namely, shear-thinning, Bingham plastic and yield shear-thinning respectively. The yield shear-thinning model is given by:

$$\tau = \tau_y + k\,\dot{\gamma}^n \qquad (2)$$

The Herschel Bulkley or the yield shear-thinning model (2) can be reduced to the Bingham plastic model when n=1 and k becomes the plastic viscosity and (2) can also be reduced to a shear-thinning model when the yield stress τ_y=0. When n=1, k becomes the Newtonian viscosity and τ_y=0, the yield shear-thinning model (2) reverts to the Newtonian model.

Haldenwang (2003) established a model for the prediction of the onset of transition in rectangular flumes. He used a semi-log plot of Re vs. Fr and noticed that for each channel inclination the shape of the curve seemed to be similar. The onset of transition was deemed to be at a point where the slope changed. A typical transition locus from 1° to 5° slope is shown in Figure 2. The critical Re for onset of transition is:

$$Re_c = 853.1\left(\mu/\mu_w \mid \dot{\gamma} = 100s^{-1}\right)^{-0.21} Fr + 12\,630\left(\mu/\mu_w \mid \dot{\gamma} = 100s^{-1}\right)^{-0.75} \qquad (3)$$

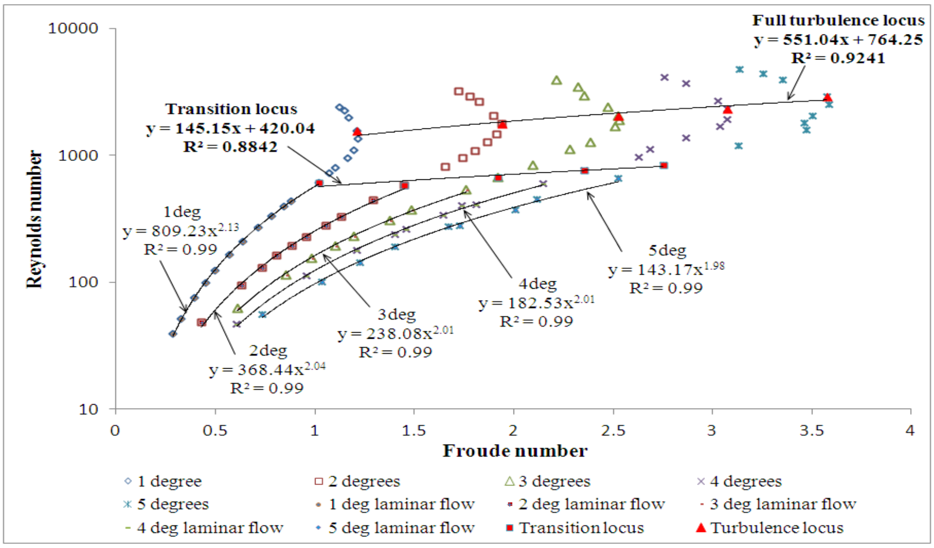


Figure 2: Onset and end of transition locus for 4% CMC suspension in a 150 mm half-round flume

The onset of “full turbulence” was deemed to be point at which the Froude number is maximum (Haldenwang, 2003). The “full turbulence” locus is also illustrated in Figure 2.

The critical Reynolds number at the onset of turbulent flow is expressed as:

$$Re_{c(turb)} = 3812\left(\mu/\mu_w \mid \dot{\gamma} = 500s^{-1}\right)^{-0.52} Fr + 9626\left(\mu/\mu_w \mid \dot{\gamma} = 500s^{-1}\right)^{-0.65} \quad (4)$$

The model which was developed by Haldenwang (2003); Haldenwang et al. (2010) for rectangular channels, will be adapted to half-round channels in this work.

Fitton (2008) developed a criterion to predict the laminar-turbulent transition region in open channels of various geometries. Thus his model is also used here.

Fitton (2008) made use of the Colebrook-White friction factor (Colebrook, 1939) for turbulent flow as well as the Darcy friction factor for laminar flow to determine the transition point which was deemed to be the intersection between the two friction factors as shown in Figure 3.

The friction factor is given by:

$$f = \frac{2\,g\,R_h \sin\theta}{V^2} \quad (5)$$

The Darcy friction factor is four times the Fanning friction factor and is given by:

$$f_D = \frac{64}{Re} \quad (6)$$

The Colebrook-White friction factor for turbulent flow in open channels is given by:

$$\frac{1}{\sqrt{4f}} = -2\log\left(\frac{e}{14.84R_h} + \frac{2.51}{Re\sqrt{4f}}\right) \qquad \textbf{(7)}$$

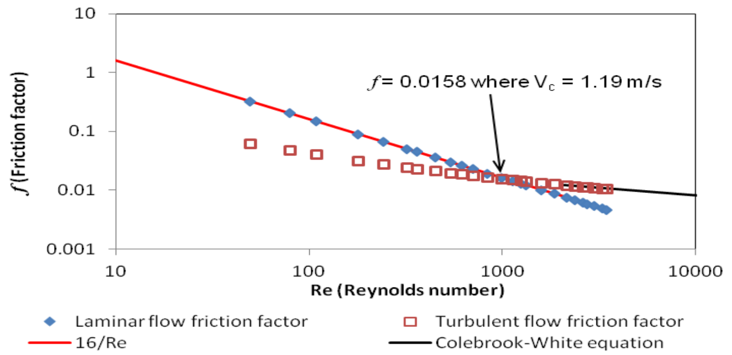


Figure 3: Prediction of transition. 7.1% kaolin in water slurry flowing in 300 mm half-round channel

Burger et al. (2010a) conducted studies in laminar flow on the shape effect of the flow of non-Newtonian fluids in open channels. They found that the laminar flow regime data can be described by a general relationship f=K/Re_H where f is defined as the Fanning friction factor, Re_H the non-Newtonian Reynolds number (Haldenwang et al., 2002) and K is a purely numerical coefficient dependent on channel shape. Among the various channel shapes studied, a K value of 16.2 was obtained for the half-round shape.

Burger et al. (2010a) found that the predicted f vs. Re line for smooth-walled half-round shaped channel to be coincident with the experimental data when plotted as a f vs. Re_H plot.

Burger (2014) modified the Blasius equation for turbulent pipe flow and established new coefficients for specific open channel shapes. For the half-round channel shape, Burger (2014) obtained a “c” value of 0.048 and a “d” value of -0.2049.

The Blasius friction factor for turbulent flow is thus given by:

$$f = 0.079\,Re^{0.25} \qquad \textbf{(8)}$$

Burger (2014) established a composite power law relationship in a channel of half-round cross-sectional shape. His model can accommodate half-round open channel data in the laminar, transition and turbulent flow regimes and is expressed as:

$$f = F_2 + \frac{(F_1 - F_2)}{\left(1 + \left(\frac{Re}{t}\right)^e\right)^f} \quad (9)$$

where F_1 and F_2 are the power law relationships given respectively by:

$$F_1 = a\,Re^b \quad (10)$$

and

$$F_2 = c\,Re^d \quad (11)$$

The coefficients of the composite power law relationship are tabulated in Table 1.

Table 1: Parameters in composite power law correlation for a half-round channel based on Re_H (Burger, 2014)

Shape	a	b	c	d	e	f	t
Half-round	16.2	-1	0.0480	-0.2049	230	0.015	1055

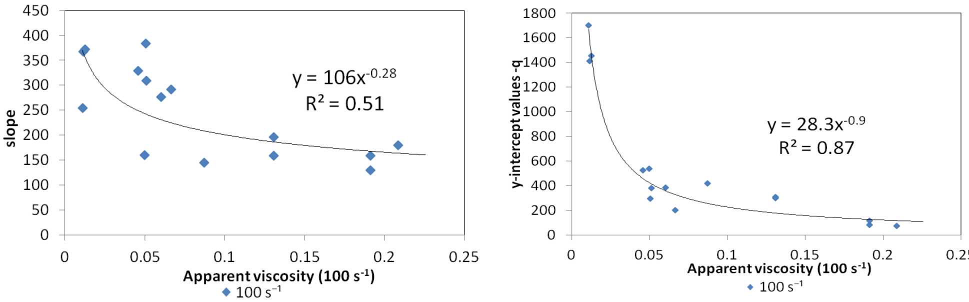
2. METHODOLOGY

2.1. ADAPTATION OF HALDENWANG'S MODEL FOR RECTANGULAR CHANNELS TO A HALF-ROUND SHAPE

A large experimental database for non-Newtonian flow produced by the Flow Process Research Centre at the Cape Peninsula University of Technology in half-round channels at slopes varying from 1° to 5° was used to achieve the objective. The test fluids consisted of bentonite (3.5 – 6.8 (% vol.)) and kaolin clay (3.4 – 9.2 (% vol.)) suspensions, and solutions of carboxymethyl cellulose (CMC) (1.5 – 5.3 (% vol.)) (Burger, 2010b). The shear stress - shear rate behaviour of each test fluid was measured using tube viscometers.

The methodology used by Haldenwang (2003) for rectangular channels was used here. A linear relationship between the Froude and the Reynolds numbers giving the slope and intercept for the viscosities at 100 s^{-1} and 500 s^{-1} was established from 1° to 5° slopes. The slope (p) and the y-intercept (q) values were plotted against the apparent viscosities at a shear rate of 100 s^{-1} for the onset of transition and a shear rate of 500 s^{-1} for the end of transition to establish the adapted critical Reynolds numbers. A shear rate of 100 s^{-1} was chosen since at that value the apparent viscosity was similar for various fluids in the region of the onset of transitional flow and a similar observation was noted at a shear rate of 500 s^{-1} for the onset of turbulent flow. Figure 4a illustrates the slope (p) against the apparent viscosity and the y-intercept (q) vs. the apparent viscosity is shown in Figure 4b for the onset of transition.

Figures 4a and 5b show the two power law relationships used to obtain the critical Reynolds number at the onset of transition in the half-round flume.



(a) (b)

Figure 4: (a) Onset of transition locus – relationship of p-values with apparent viscosity at 100 s^{-1} for all fluids flowing in 150 and 300 mm half-round flumes (b) relationship of q-values with apparent viscosity at 100 s^{-1} corresponding to a.

Thus, the adapted critical Reynolds number for the half-round flume is written as:

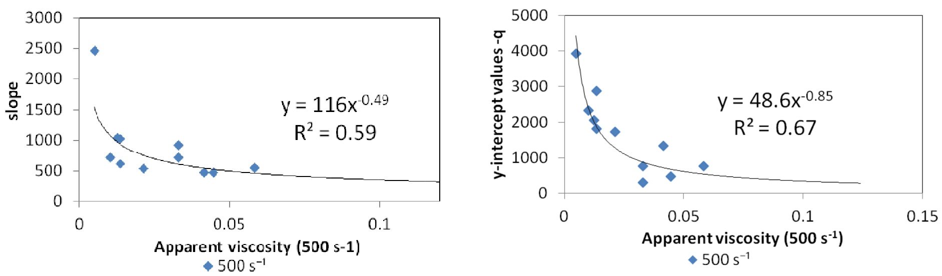
$$Re_c = 106\left(\mu \mid \dot{\gamma} = 100s^{-1}\right)^{-0.28} Fr + 28\left(\mu \mid \dot{\gamma} = 100s^{-1}\right)^{-0.9} \quad (12)$$

The constant 106 has a unit of $Pa.s^{0.28}$ and the constant 28, a unit of $Pa.s^{0.9}$.

The dimensionless form of the critical Reynolds number for the onset of transition in a half-round flume (Equation (11)) is expressed as:

$$Re_c = 733\left(\mu/\mu_w \mid \dot{\gamma} = 100s^{-1}\right)^{-0.28} Fr + 14\,033\left(\mu/\mu_w \mid \dot{\gamma} = 100s^{-1}\right)^{-0.9} \quad (13)$$

Similarly, the critical Reynolds number for the onset of turbulent flow in the half-round flume was obtained from Figures 5a and 5b.



(a) (b)

Figure 5: (a) Onset of 'Full turbulence' locus – relationship of p-values with apparent viscosity at 500 s^{-1} for all fluids flowing in 150 and 300 mm half-round flumes (b) relationship of y-intercept values with apparent viscosity at 500 s^{-1} corresponding to a.

The half-round flume upper critical Reynolds number is written as:

$$Re_c = 116\left(\mu \mid \dot{\gamma} = 500s^{-1}\right)^{-0.49} Fr + 49\left(\mu \mid \dot{\gamma} = 500s^{-1}\right)^{-0.85} \quad (14)$$

The constant 116 has a unit of Pa.s$^{0.49}$ and the constant 49, a unit of Pa.s$^{0.85}$.

The dimensionless form of Equation (13) is expressed as:

$$Re_{c(turb)} = 3423\left(\mu/\mu_w \mid \dot{\gamma} = 500s^{-1}\right)^{-0.49} Fr + 17\,386\left(\mu/\mu_w \mid \dot{\gamma} = 500s^{-1}\right)^{-0.85} \quad (15)$$

The method used to calculate the critical Reynolds numbers in rectangular channels was described by Haldenwang (2003). Thus the adaptation to the half-round channel is briefly described here. The calculation of the onset of transition is shown in Figure 6a and the end of transition is illustrated in Figure 6b. The turbulent velocity is given by:

$$V = \sqrt{g\,h \sin\alpha}\left(2.5\ln\frac{2R_h}{k} - 76.86\,\mu_{app500} - 9.45\right) \quad (16)$$

where the Fanning friction factor is given by

$$f_{turb} = 0.66\left(\frac{2\,g\,h\sin\alpha}{V^2}\right) \quad (17)$$

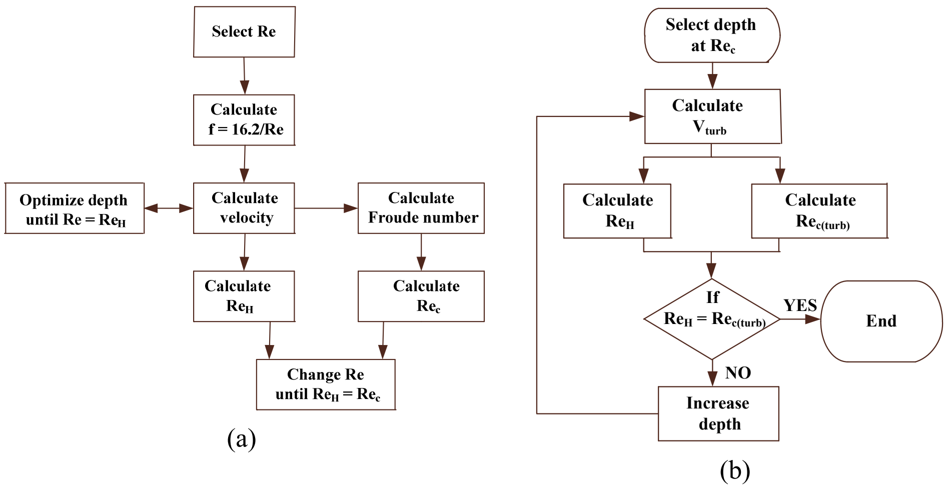


Figure 6: Flow chart for the calculation of transition. (a) Onset of transition (b) Onset of "full turbulence" (Haldenwang, 2003)

Once the friction factor and the Reynolds numbers were obtained at the onsets of transition and "full turbulence", a power law relation was established between the two points as shown in Figure 7.

The half-round models for transition (Equations (13) and (15)) were evaluated for a 5.3% CMC in water suspension flowing in a 150 mm half-round flume and the fit is shown in Figure 7.

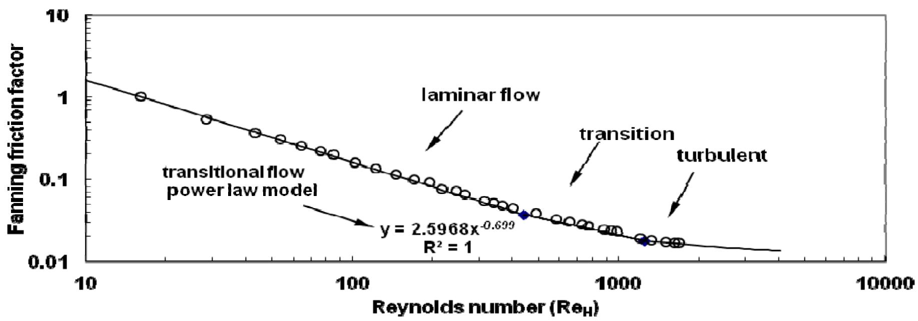
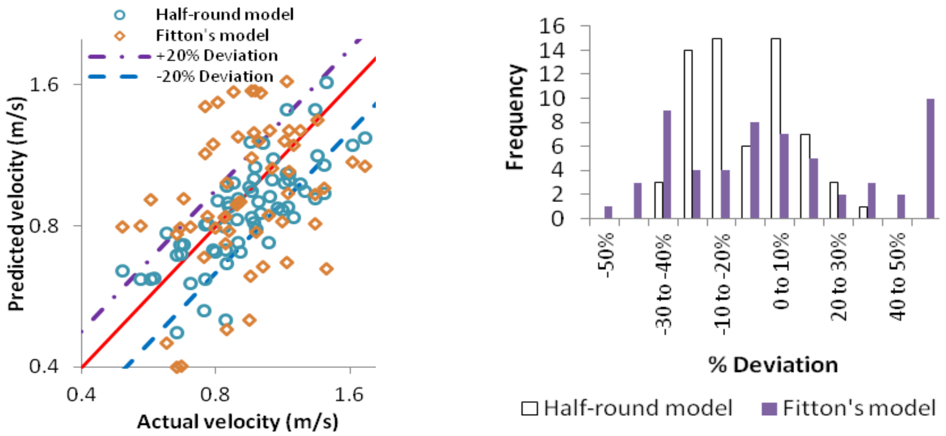


Figure 7: Evaluation of the transition models for the half-round flume

3. RESULTS

3.1. ONSET OF TRANSITIONAL FLOW

A comparison is made between Fitton's model and the adapted critical Reynolds number for half-round channels based on Haldenwang's (2003) model. It can be seen from Figure 8a (Table 2)



a **b**

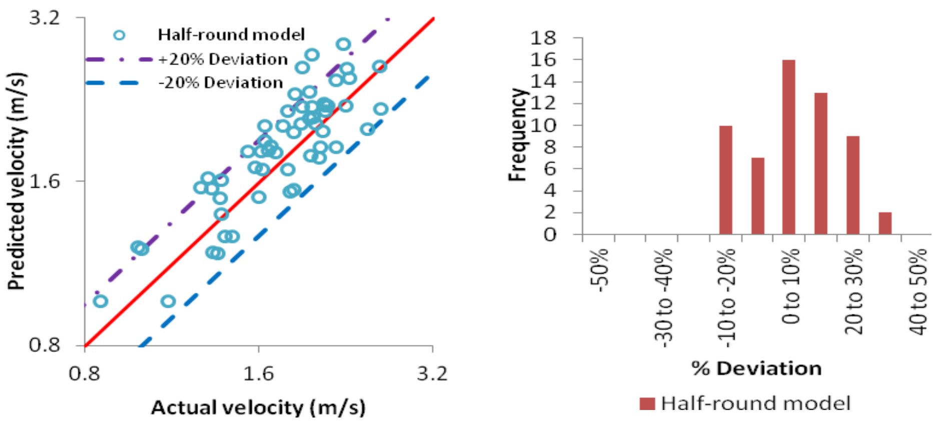
Figure 8: (a) Model comparison for the onset of transition in a half-round flume. (b) Frequency vs. % Deviation of transition data points corresponding to (a).

Table 2: Statistical analysis for the onset of transition in a half-round flume

	LSE	Min% Dev	Max % Dev	Standard deviation	% Data falling within the 20% margin
Re_H Half-round	0.0110	-40	30	0.24	67
Re Fitton	0.0205	-54	90	0.32	41

3.2. END OF TRANSITIONAL FLOW

Figure 9a shows the prediction of the half-round model for the onset of turbulent flow. It can be seen that the adapted half-round model model gives a good prediction of the end of transition since three quarters of the data points were within the +/- 20% deviation range as suggested in Table 3. Figure 9b also suggests that the half-round model gives a good prediction. The distribution shown in Figure 9b indicates that the half-round model is positively skewed. No comparison with other models was made due to the lack thereof in literature.



a **b**

Figure 9: (a) Model comparison for the end of transition in a half-round flume. (b) Frequency vs. % Deviation of transition data points corresponding to (a).

Table 3: Statistical analysis for the end of transition in a half-round flume

	LSE	Min% Dev	Max % Dev	Standard deviation	% Data falling within the 20% margin
Re_H Half-round	0.0085	-19	39	0.46	81

4. CONCLUSIONS

A new transition model for flow of non-Newtonian fluids in half-round channels based on the Haldenwang (2003) model for rectangular channels was established. The model was compared to the Fitton (2008) model and performed better. The adapted model offers an advantage over Fitton's (2008) model since the onset of "full turbulence" can also be predicted. The adapted model for the onset of "full turbulence" could not be compared since it is the only model currently available in literature. Different models will be adapted from the Haldenwang (2003) method in channels of triangular and trapezoidal shapes and it will be attempted to establish a composite model which can be used to predict transition in channels of triangular, half-round, trapezoidal and rectangular shapes. This will be published at later stage.

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